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Contents

Invited talks

Sparse Interpolation and Exponential Analysis

Annie Cuyt
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Sparse Interpolation (SI) from computer algebra and Exponential Analysis (EA) from digital signal processing are both actually inverse problems. The goal is to identify and reconstruct a sparse linear combination of monomials or a sparse linear combination of exponential functions. Hence SI and EA are also connected to polynomial interpolation and Fourier analysis respectively.

We discuss how SI and EA can cross-fertilize and lead to new results in the solution of several hard problems, such as the numerical reconditioning of the inverse problem, the separation of clustered exponential frequencies, retrieval of the correct number of terms in the linear combination, a divide and conquer algorithm for the inverse problems, and last but not least superresolution.

Persistent homology applications in data analysis

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In recent years, in the context of *persistent homology* to deal with *persistence diagrams* in supervised learning approaches various kernels have been proposed. In this talk, after recalling some basic notions of toplogical data analys, persistent homology, persistent diagrams, we present two applications of persistent homology:

- *Variably Scaled Persistence Kernels (VSPK)* for classification with support vector machine [2, 3];
- Estimate the *Intrinsic Dimension (ID)* of point clouds [1].

We also show some useful examples and numerical tests.

[1] C. Bandiziiol, S. De Marchi, M. Allega: On computing the intrinsic dimension of point clouds by a persistent homology approach, submitted.

[2] C. Bandiziiol, S. De Marchi: Persistence Symmetric Kernels for Classification: A Comparative Study. *Symmetry* **2024**, *16, 1236*

[3] De Marchi S., Lot F., Marchetti F., Poggiali D.: Variably Scaled Persistence Kernels (VSPKs) for persistent homology applications. *Journal of Computational Mathematics and Data Science* **2022**, *4, 100050*

Adaptive hierarchical spline approximation: the perspective of isogeometric methods

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The quality of geometric representations is crucial for accurate numerical simulations. Despite significant advances in modeling tools, mesh generation and integration between Computer-Aided Design (CAD) and Computer-Aided Engineering (CAE) remain persistent challenges in scientific computing. Isogeometric Analysis (IGA) addresses these issues by promoting a unified spline-based representation that bridges the gap between design and analysis. Beyond geometric modeling, the inherent smoothness of spline functions offers key numerical advantages, including enhanced accuracy and robustness. However, traditional tensor-product spline models limit local mesh refinement and geometric flexibility, restricting their applicability to complex CAD/CAE scenarios. Recent developments in adaptive spline technologies overcome these limitations by enabling localized refinement while preserving the smoothness and accuracy inherent to IGA. In this context, the mathematical foundations of adaptive isogeometric methods—crucial for problems requiring highly refined meshes—have been recently established [1] and extended to the multi-patch setting [2].

This talk will explore how these advances extend the capabilities of isogeometric methods, paving the way for more robust simulation pipelines and alternative spline refinement solutions. Applications include phase-field modeling of evolving interface problems and computational mechanics challenges inspired by additive manufacturing, see, e.g., [3].

[1] A. Buffa, G. Gantner, C. Giannelli, D. Praetorius, R. Vázquez. *Mathematical foundations of adaptive isogeometric analysis*, Arch. Comput. Method E. (2022) 29, 4479–4555.

[2] C. Bracco, C. Giannelli, M. Kapl, R. Vázquez. *Adaptive isogeometric methods with C^1 (truncated) hierarchical splines on planar multi-patch domains*, Math. Mod Meth. Appl. S. (2023) 33, 1829–1874.

[3] M. Carraturo, M. Torre, C. Giannelli, A. Reali. *An isogeometric approach to coupled thermomechanics in 3D via hierarchical adaptivity*, Comput. Math. Appl. (2024) 133–144.

When design meets reuse: Geometric and structural adaptation of grid shells via differentiable optimization and geometric deep learning

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Architectural Geometry is an interdisciplinary field that brings together mathematics, architectural design, and computational methods to address the challenges of designing and shaping complex structures. In this talk, I introduce a computational framework that harnesses differentiable optimization and geometric deep learning to design 3D free-form architectural surfaces: specifically, grid shells constructed from reclaimed elements sourced from dismantled buildings. The aim is to sustain circular construction practices that minimize waste and energy consumption. This setting introduces a combinatorial and geometric challenge: how can a new design be adapted to fit a stock of reclaimed elements, while preserving the intended design? Our approach combines discrete assignment of inventory elements to maximize reuse and minimize new fabrication, with continuous differentiable optimization to reduce cut-off waste and maintain geometric fidelity. In addition, to improve structural performance, we incorporate a neural module that adaptively reconfigures the shape of the structure in a self-supervised manner, without the need for pre-existing training data. Unlike conventional methods that adapt forms to materials, our system begins with user-defined shapes and adjusts them only when necessary, preserving design intent. This allows for the integration of sustainability and structural efficiency directly into the design process, thus offering a well-founded pathway for circular construction practices.

On Pythagorean-hodograph curves and their applications

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Polynomial Pythagorean-hodograph (PH) curves ([1]), characterized by the property that their unit tangent vectors are rational, offer numerous advantages for practical applications. Planar PH curves are important since they posses rational offset curves. Spatial PH curves are especially interesting because their construction from a quaternion preimage curve allows one to equip the curve with a rational orthonormal adapted frame which makes these curves highly effective for motion design applications. Another advantage of the polynomial PH curve is that its arc length function is also a polynomial. This property significantly simplifies the computation of PH curves with prescribed length and facilitates the development of efficient real-time interpolator algorithms, making PH curves especially useful in robotics. Extending the Pythagorean condition to Minkowski space leads to Minkowski Pythagorean-hodograph (MPH) curves, which provide a powerful tool for deriving rational parameterizations of planar domains via the medial axis transform.

In the talk, recent developments on the construction of PH and MPH curves are presented. The first part focuses on algorithms for computing curves that geometrically interpolate given data, have a prescribed arc length, and support fully local construction of G^2-continuous (M)PH splines. A key advantage of the proposed methods is that the existence of interpolants is guaranteed for any input interpolation data, which is generally not the case due to the nonlinear nature of these curves. In the second part, the L^2 approximation using spatial PH curves is discussed, which requires the solution of a complicated non-linear optimization problem. As an alternative, a simpler construction based on the L^2 approximation in the preimage space is suggested. The effectiveness of the presented algorithms is demonstrated through asymptotic analysis and several numerical examples.

[1] R. T. Farouki, Pythagorean-Hodograph Curves: Algebra and Geometry Inseparable, Springer, Berlin (2008).

Adaptive FEEC with hierarchical splines

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Finite Element Exterior Calculus (FEEC) is a mathematical framework that integrates finite element methods with concepts from differential geometry and algebraic topology [1]. It provides a systematic approach to discretizing partial differential equations, ensuring stability and compatibility across various applications in computational electromagnetism and fluid mechanics. This talk will focus on isogoemetric version of FEEC and will present some recent developments in the theory and application of adaptive structure-preserving discretizations.

In particular, we will present theoretical conditions under which Truncated Hierarchical B-splines (THB-splines) allow for the construction of an exact discrete de Rham complex [3]. We will also discuss practical approaches that help satisfy those theoretical conditions when performing adaptivity [4]. We will also present some example applications to fluid flow and magnetohdrodynamics simulations [5]. Finally, we will show extensions that go beyond the THB-spline de Rham complex.

[1] Arnold, D.N., Falk, R.S. and Winther, R., 2006. Finite element exterior calculus, homological techniques, and applications. Acta numerica, 15, pp.1-155.

[2] Giannelli, C., Jüttler, B. and Speleers, H., 2012. THB-splines: The truncated basis for hierarchical splines. Computer Aided Geometric Design, 29(7), pp.485-498.

[3] Shepherd, K. and Toshniwal, D., 2024. Locally-Verifiable Sufficient Conditions for Exactness of the Hierarchical B-spline Discrete de Rham Complex in $\mathbb{R}^n$. Foundations of Computational Mathematics, pp.1-43.

[4] Cabanas, D.C., Shepherd, K.M., Toshniwal, D. and Vázquez, R., 2025. Construction of exact refinements for the two-dimensional HB/THB-spline de Rham complex. arXiv preprint arXiv:2502.19542.

[5] Zhang, Y., Palha, A., Brugnoli, A., Toshniwal, D. and Gerritsma, M., 2024. Decoupled structure-preserving discretization of incompressible MHD equations with general boundary conditions. arXiv preprint arXiv:2410.23973.

Quasi-interpolatory subdivision schemes generalizing the cubic B-spline and fourth-point schemes

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The four-point interpolatory scheme and the cubic B-spline are examples of the most well-known subdivision techniques, each with its own strengths and weaknesses. In this study, we present several quasi-interpolatory subdivision schemes that aim to combine the strengths of both methods. In particular, we present a modified butterfly subdivision scheme over regular triangular meshes. The proposed technique is an approximating scheme with a tension parameter. It achieves fourth-order accuracy and C^2 smoothness for a suitable range of the parameter while maintaining the same support of the original butterfly scheme. Additionally, we present a novel class of shape preserving C^2 subdivision schemes with third-order accuracy, preserving both monotonicity and convexity of the given data, regardless of the issue of strictness and non-strictness. To achieve this, we especially introduce a modified *minmod* method that plays a role of limiting procedure to prevent spurious oscillations. Some numerical results are presented to demonstrate the performance of the proposed schemes.

Mini-symposia

Spline quasi- and quasi^2- interpolating projectors for the numerical solution of Cauchy singular Fredholm integral equations

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joint work with Mattia Alex Leoni, Sara Remogna

Following recent studies [1, 2] related to the approximation of weakly singular linear and nonlinear Fredholm integral equations of the second kind by means of collocation and Kulkarni's methods, also in their iterated versions, based on spline quasi-interpolating projectors, we analyse in the same framework the case of integral operators with Cauchy (strongly) singular kernel [3]. This feature compels us to modify the previously considered spline quasi-interpolating projectors or to use their quasi^2-interpolating variant, within the sole collocation approach. Several numerical results validate the proposed error estimates.

[1] A. Aimi, M.A. Leoni, S. Remogna, Numerical solution of Fredholm integral equations by quasi-interpolating projectors, In: D. Sbibih, S. Remogna, A. Serghini (Eds.) New Trends in Shape Modelling and Approximation Methods. MACMAS 2023. *SEMA SIMAI Springer Series*, Springer, 37 (2024) 109–122.

[2] A. Aimi, M.A. Leoni, S. Remogna, Numerical solution of nonlinear Fredholm-Hammerstein integral equations with logarithmic kernel by spline quasi-interpolating projectors, *Math. Comput. Simul.*, 223 (2024) 183–194.

[3] A. Aimi, M.A. Leoni, S. Remogna, Spline quasi-interpolating and quasi^2-interpolating projectors for the numerical solution of Cauchy singular integral equations, submitted (2025)

A smoothly varying quadrature technique for IgABEM in Stokes flow simulations

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joint work with Francesco Patrizi, Alessandra Sestini

Isogeometric boundary elements methods typically involve the numerical evaluation of singular integrals, which, in classical approaches are classified as weakly singular, nearly singular, or regular [1,2,3]. We propose a new technique where the classification is reduced to just two cases: (weakly) singular and non-singular. We obtain this goal by defining a smoothly varying quadrature rule which automatically changes based on the physical distance from singularities of the integral kernels, in order to improve accuracy and efficiency. Moreover, the rule provides integration on B-spline supports rather than on single elements, therefore reducing computational cost (especially for higher-degrees). We highlight the features of this approach by showing examples of its application in the numerical solution of Stokes flow equations.

[1] A. Aimi, F. Calbrò, A. Falini, M.L. Sampoli, A. Sestini, Quadrature formulas based on spline quasi-interpolation for hypersingular integrals arising in IgA-SGBEM, *Comput. Method Appl. M.* 372 (2020).

[2] M. Harmel, R. A. Sauer, New hybrid quadrature schemes for weakly singular kernels applied to isogeometric boundary elements for 3D Stokes flow, *Eng. Anal. Bound. Elem.* 153 (2023), 172-200.

[3] L. Heltai, Nonsingular isogeometric boundary element method for Stokes flows in 3D, *Comput. Method Appl. M.* 268 (2014), 514-539.

Error bounds for numerical differentiation with polyharmonic radial basis functions

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In the local setting of numerical differentiation standard error bounds for polyharmonic and thin-plate spline RBF show convergence orders weaker than those expected from the polynomial exactness degree due to the polynomial term appended to these conditionally positive definite functions. Yet, proofs based on the polynomial reproduction alone do not explain why the RBF term is at all useful. I will present new error estimates that fill this gap.

Numerical integration of nearly singular integrals in 2D IGA-BEM

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Singularity extraction is a common prerequisite in Boundary Element Method (BEM) to evaluate singular integrals by separating the singular part of the singular integral from the regular part. The separation is obtained by truncating a series expansion of the singular kernel at the singular point. The singular part is then computed analytically, while a suitable quadrature rule is applied on the regular part.

Nearly singular integrals are regular integrals, but with a very strong oscillation due to a nearby presence of the singularity (e.g., when the singular point is very near the integration domain). For line integrals, the singularity extraction was generalized to nearly singular kernels by writing the expansion about the nearby complex points [1].

In this talk we present this idea in the isogeometric framework to model problems on non-smooth 2D domains. Applying a quasi-interpolation-based quadrature rule [2], we demonstrate on numerical examples that the expected optimal orders of convergence for the approximate solutions of the given boundary value problems are achieved with a small number of uniformly spaced quadrature nodes.

[1] L. Klinteberg and A.H. Barnett, Accurate quadrature of nearly singular line integrals in two and three dimensions by singularity swapping, BIT (2021), 83–118.

[2] F. Mazzia, A. Sestini, A., Quadrature formulas descending from BS Hermite spline quasi-interpolation, J. Comput. Appl. Math. (2012) 236, 4105–4118.

B-spline Hermite Quasi Interpolants: from Differential Equations Dense Output to Data Mining Applications

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The talk is devoted to a review of the applications of the BS Hermite quasi-interpolation schemes (BSHQIs). The BSHQIs have been derived as continuous extensions of BS-methods for the solution of Differential Equations. They have been successfully employed as continuous extensions for finite difference and Runge-Kutta methods, on non-uniform meshes and for the approximation of finite one-dimensional integrals. In the context of the construction of the continuous parametrization of planar domains, BSHQIs are proposed as final continuous models, ensuring the desired smoothness. Due to their properties, they have been used for the preprocessing of univariate time series (imputation and smoothing), subsequently being applied in forecasting and anomaly detection, also through the use of dynamic copulas. Furthermore, they have been used for univariate density estimation to correctly identify the underlying data distribution, and applied in the context of clustering modeling using copulas. The BSHQI has also been applied to multispectral images demosaicing, in collaboration with Planetek Italia s.r.l.. Some of these methodologies have been developed within the PNRR project Future Artificial Intelligence Research (FAIR).

[1] Tamborrino C., Falini A., Mazzia F. *Empirical density estimation based on spline quasi-interpolation with applications to copulas clustering modeling* (2024) Journal of Computational and Applied Mathematics, 452, art. no. 116131,

[2] Falini A., Mazzia F., Sestini A. *Hermite–Birkhoff spline Quasi-Interpolation with application as dense output for Gauss–Legendre and Gauss–Lobatto Runge–Kutta schemes* (2024) Applied Numerical Mathematics, 200, pp. 273 - 285,

[3] Falini A., Mazzia F., Tamborrino C. *Spline Based Hermite Quasi-Interpolation for univariate Time Series* (2022) Discrete and Continuous Dynamical Systems - Series S, 15 (12), pp. 3667 - 3688,

[4] Mazzia F., Sestini A. *The BS class of Hermite spline quasi-interpolants on nonuniform knot distributions* (2009) BIT Numerical Mathematics, 49 (3), pp. 611 - 628,

Application of a metric for complex polynomials to bounded modification of planar Pythagorean–hodograph curves

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joint work with Rida T. Farouki, Marjeta Knez, Vito Vitrih

By interpreting planar polynomial curves as complex–valued functions of a real parameter, an inner product, norm, metric function, and the notion of orthogonality may be defined for such curves. This approach is applied to the complex pre–image polynomials that generate planar Pythagorean–hodograph (PH) curves, to facilitate the implementation of bounded modifications of them that preserve their PH nature. The problems of bounded modifications under the constraint of fixed curve end points and end tangent directions, and of increasing the arc length of a PH curve by a prescribed amount will also be addressed.

Regularization Methods for Planar Offset Curves

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joint work with Mariantonia Cotronei, Domenico Fazzino, Tomas Sauer

The construction of offset curves for planar trajectories is a fundamental problem in several applications, including the generation of tool paths for numerically controlled industrial machines and the development of trajectory planning methods for autonomous driving systems. While the approximation of planar curves using polynomial splines of arbitrary degree under geometric continuity constraints has been widely studied [1,2], the computation of accurate and well-behaved offset curves remains a challenging task. In particular, offset curves typically do not belong to the same functional space as the original trajectory. For example, the offset of a spline curve is not generally a spline itself. Moreover, offset curves may exhibit peculiar singularities, including self-intersections, which complicate their use in practical applications. Existing approaches in the literature address these issue through geometric filtering techniques to detect and remove undesirable features [3]. In [4], a Hermite interpolation scheme is proposed to approximate offset curves when the original trajectory is a polynomial curve, producing a polynomial representation of the offset. However, such direct approximations often fail to preserve essential characteristics of the original curve, due to the inherent nonlinearity of the offsetting operation, which involves a displacement along the normal vector field of the curve. In this work, we present a regularization technique based on Hermite spline regression [5], which incorporates both function values and derivative information, enabling the simultaneous fitting of positions and tangents. The regularization is imposed on the second derivatives, effectively mitigating the jerk effect, which is particularly relevant in motion planning and path smoothing applications. Additionally, the contribution of the first derivative in the fitting process is modulated by a weight parameter, which is heuristically determined to balance adherence to tangent data against overall offset curve smoothness.

[1] Josef Hoschek, Offset curves in the plane, Computer-Aided Design, Volume 17, Issue 2, 1985, Pages 77-82, ISSN 0010-4485.

[2] Josef Hoschek, Spline approximation of offset curves, Computer Aided Geometric Design, Volume 5, Issue 1, 1988, Pages 33-40, ISSN 0167-8396.

[3] Hongyan Zhao, Lian Zhou (2017). A Novel Method of Offset Approximation along the Normal Direction with High Precision. Mathematical and Computational Applications, 22(3), 39.

[4] Young Joon Ahn. Hermite interpolation of offset curves of parametric regular curves. AIMS Mathematics, 2023, 8(12): 31008-31021, doi: 10.3934/math.20231587

[5] Rosanna Campagna, Mariantonia Cotronei, Domenico Fazzino, A Hermite spline model for data regression (2025) Mathematics and Computers in Simulation, 229, pp.222 - 234, DOI: 10.1016/j.matcom.2024.09.011

Rebricking frames and bases

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joint work with Thomas Fink, Florian Heinrich, Moritz Proell

In 1946, Dennis Gabor introduced the analytic signal $f + iHf$ for real-valued signals f. Here, H is the Hilbert transform. This complexification of functions allows for an analysis of their amplitude and phase information and has ever since given well-interpretable insight into the properties of the signals over time. The idea of complexification has been reconsidered with regard to many aspects: Examples are the dual tree complex wavelet transform, or via the Riesz transform and the monogenic signal, i.e. a multi-dimensional version of the Hilbert transform, which in combination with multi-resolution approaches leads to Riesz wavelets, and others. In this context, we ask two questions:

- Which pairs of real orthonormal bases, Riesz bases, frames and Parseval frames $\{f_n\}_{n\in\mathbb{N}}$ and $\{g_n\}_{n\in\mathbb{N}}$ can be "rebricked" to complex-valued ones $\{f_n + ig_n\}_{n\in\mathbb{N}}$?
- And which real operators A allow for rebricking via the ansatz $\{f_n+iAf_n\}_{n\in\mathbb{N}}$?

In this talk, we give answers to these questions with regard to a characterization which linear operators A are suitable for rebricking while maintaining the structure of the original real valued family. Surprisingly, the Hilbert transform is not among them.

[1] D. Gabor, Theory of communication, Trans. Inst. Electr. Eng. 3 (1946) 429–456.

[2] T. Fink, B. Forster, F. Heinrich, Rebricking Frames and Bases. J. Math. Anal. Appl. 544 (2025) 129051

[3] T. Fink, B. Forster-Heinlein, F. Heinrich, M. Proell: Examples of Rebricking. Trends in Mathematics, Research Perspectives. Birkhäuser. In press.

Efficient Clustering on Riemannian Manifolds by Fréchet Mappings

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joint work with Robert Azencott, Nicolas Charon, Andreas Mang, Ji Shi

Symmetric Positive Definite (SPD) matrices, in particular, correlation matrices, appear in several applications including high-dimensional statistics, communication networks and image analysis. Although SPD matrices do not form a vector subspace of the Euclidean space, they possesses a smooth manifold structure that can be endowed with a Riemannian metric. As a result, the most meaningful way to measure similarity between SPD matrices is through a Riemannian metric rather than Euclidean distance.

This work is motivated by brain imaging applications, where correlation matrices derived from functional Magnetic Resonance Imaging (fMRI) data are employed to model the strength of neural connections between various brain sites and to evaluate brain function. These matrices are typically large (e.g., 2000×2000), and even small subject cohorts result in significant computational demands when applying clustering techniques to extract meaningful patterns. To manage these challenges, existing methods often disregard the underlying manifold structure or reduce the rank of the matrices - both of which can lead to information loss.

To overcome these computational challenges, we propose a novel algorithmic framework for efficient clustering on a class of Riemannian manifolds that includes the space of SPD matrices. The core innovation of our method is the use of a Fréchet mapping, which substantially reduces computational complexity.

In this talk, I will present the mathematical foundation of the proposed approach and demonstrate its effectiveness through numerical examples, including applications to real fMRI datasets.

ESPIRA: Estimation of Signal Parameters by Iterative Rational Approximation

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joint work with Nadiia Derevianko, Markus Petz

We consider exponential sums of the form

$$f(t) = \sum_{j=1}^{M} \gamma_j \, \mathrm{e}^{\phi_j t} = \sum_{j=1}^{M} \gamma_j \, z_j^t,$$

where $M \in \mathbb{N}$, $\gamma_j \in \mathbb{C}\setminus\{0\}$, and $z_j = \mathrm{e}^{\phi_j} \in \mathbb{C}\setminus\{0\}$ with $\phi_j \in \mathbb{C}$ are pairwise distinct. The recovery of such exponential sums from a finite set of possibly corrupted signal samples plays an important role in many signal processing applications, see e.g. in phase retrieval, signal approximation, sparse deconvolution in nondestructive testing, model reduction in system theory, direction of arrival estimation, exponential data fitting, or reconstruction of signals with finite rate of innovation.

We present a new method for **E**stimation of **S**ignal **P**arameters based on **I**terative **R**ational **A**pproximation (ESPIRA) for sparse exponential sums. Our algorithm uses the AAA algorithm for rational approximation of the discrete Fourier transform of the given equidistant signal values. We show that ESPIRA can be interpreted as a matrix pencil method applied to Loewner matrices. These Loewner matrices are closely connected with the Hankel matrices which are usually employed for recovery of sparse exponential sums. ESPIRA achieves similar recovery results for exact data as ESPRIT and the matrix pencil method (MPM) but with less computational effort. Moreover, ESPIRA strongly outperforms ESPRIT and MPM for noisy data and for signal approximation by short exponential sums.

Notes on Polynomials with falling and raising factorials

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The relation between the polynomial reproduction of interpolatory subdivision operators S_a with a finitely supported mask $a \in \ell_{00}(\mathbb{Z}^s)$ with expanding diagonal matrices $M \in \mathbb{Z}^s$ and the corresponding symbols $a^\sharp$ are well investigated. We now take a closer look at the relation to the corresponding subsymbols. In doing so, we come across an interesting relation between polynomials and products of falling and raising factorials. Prescisely, for $1_s \le \xi \le \kappa - 1_s$ and $x \in \mathbb{N}^s$ one has

$$\sum_{0 \le \beta \le \xi} \binom{\kappa}{\beta} (-1_s)^{\kappa-\beta} \; (x)^{\overline{\kappa-\beta}} \; (x)^{\underline{\beta}} \; = \; (x)^{\overline{\kappa-\xi}} \; \binom{\kappa - 1_s}{\xi} (-1_s)^{\kappa-\xi} \prod_{d=1}^{s} \prod_{\ell_d=1}^{\xi_d} (x_d - \ell_d)$$

which will be explained together with its relevance in polynomial reproduction by subdivision operators.

Multidimensional Scaling Functions and Scaling Filters with arbitrary Scaling Matrices

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The concept of multiresolution analysis as it was introduced by Mallat enables us to unterstand the link between scaling functions, wavelets, and a corresponding set of discrete filters, making it possible to compute a discrete wavelet decomposition through the application of a filterbank. For the one-dimensional dyadic situation, Mallat and Meyer established a set of filter properties that allow to decide if a given filter yields a feasible scaling function and therefore a multiresolution analysis. The corresponding theorem can be transferred to arbitrary dimensions and the use of arbitrary scaling matrices. However, in this process some complexities occur. In this talk we want to focus on one particular part of the proof, where an integral and its partition play an decisive role and some small observations about lattices serve a functional purpose.

[1] Stephane Mallat. *A Wavelet Tour of Signal Processing - The Sparse Way.* Third Edition. Amsterdam, Boston: Academic Press, 2008.

[2] Eric Viscito and Jan P. Allebach. "The Analysis and Design of Multidimensional FIR Perfect Reconstruction Filter Banks for Arbitrary Sampling Lattices" In: *IEEE Transactions on Circuits and Systems* 38.1 (1991), pp. 29-41

Rational Powell–Sabin B-splines in isogeometric methods

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joint work with Ada Šadl Praprotnik, Hendrik Speleers

Polynomial splines on unstructured triangulations in finite element theory have been traditionally characterized by interpolation problems such that each polynomial piece is determined by local interpolation data. To align these techniques with the concepts of isogeometric analysis, one approach is to express finite elements in the Bernstein–Bezier representation. A further advance in this direction is the development of globally defined locally supported basis functions on triangulations that form a convex partition of unity, thereby mimicking the properties of tensor product B-splines. One obvious benefit of this framework is the possibility to define well-behaved rational basis functions on unstructured partitions.

In this contribution we review the techniques for constructing B-spline-like functions on Powell–Sabin refined triangulations and consider their natural extension to rational forms by assigning a positive weight to each basis function. We discuss domain parametrization methods and conversion from NURBS. In addition, we present numerical examples of solving boundary value problems on planar and surface domains and demonstrate options for local refinement.

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[2] J. Grošelj and H. Speleers, Extraction and application of super-smooth cubic B-splines over triangulations. Comput. Aided Geom. Des. (2023) 103.

Isogeometric multigrid methods for G^1 multi-patch domains

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joint work with Stefan Takacs, Thomas Takacs

This talk discusses the development of multigrid solvers for biharmonic problems discretized using isogeometric analysis (IGA), with a focus on C^1-smooth multi-patch domains. These discretizations are used in the modeling of thin plates and shells. A central aspect is the nestedness of the underlying function spaces, which is crucial for constructing prolongation and restriction operators in the multigrid framework.

We consider analysis-suitable G^1 multi-patch parameterizations, cf. [1,3], which ensure C^1 continuity across patch interfaces while preserving a hierarchical structure suitable for multigrid. This nested structure enables consistent refinement and efficient implementation of multigrid components.

Extending the framework introduced in [4], we examine the structure of smoothing operators under such spaces and study two-level refinement relations that respect the geometry and continuity constraints. The approach is also aimed at generalizing to arbitrary C^1-smooth multi-patch surfaces [2].

Numerical experiments on a two-patch domain are used to assess solver performance. This work is part of the project "Isogeometric multi-patch shells and multigrid solvers".

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Analysis-suitable parameterization for isogeometric analysis

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joint work with Matthias Möller

In isogeometric analysis (IGA), generating high-quality parameterizations from CAD boundary representations is crucial for accurate and efficient simulation. This talk presents a suite of analysis-suitable parameterization techniques developed to bridge the gap between CAD and CAE, enabling seamless simulation workflows in complex domains.

We recall optimization- [1,2] and PDE-based [3] methods for generating bijective and low-distortion parameterizations, with particular emphasis on robust strategies for elongated and highly anisotropic domains. Key innovations include barrier and penalty function-based formulations, Jacobian regularization, and adaptive techniques that respond to geometric and physical field features.

In addition, we introduce a curvature-driven anisotropic parameterization strategy [4], which interprets IGA solutions as parametric surfaces and leverages principal curvatures to guide mesh density and directionality. This leads to improved accuracy in localized feature regions and greater efficiency in downstream simulations.

The proposed techniques have been successfully applied to challenging industrial geometries, such as twin-screw compressors [5], and validated using commercial CFD software. Core implementations are available as open-source code in the G+Smo library, supporting further research and practical deployment.

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A positive partition of unity basis for the Alfeld split

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joint work with Jean-Louis Merien, Hendrik Speleers

The Alfeld split of a simplex in $\mathbb{R}^s$, $s \geq 2$ is obtained by subdividing the simplex into $s+1$ subsimplices using the barycenter as a split point. On this split we consider a C^1 spline space of any degree $d \geq s+1$ and construct a positive partition of unity basis of multivariate B-splines, known as simplex splines, for the space. A Marsden-like identity and explicit C^1 smoothness conditions across a facet between two simplices are shown.

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Isogeometric discrete differential forms with Tchebycheffian B-splines

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joint work with H. Speleers, D. Toshniwal

Tchebycheffian splines are smooth piecewise functions whose pieces are drawn from (possibly different) Tchebycheff spaces, a natural generalization of algebraic polynomial spaces. They enjoy most of the properties known in the polynomial spline case. Under suitable assumptions, Tchebycheffian splines admit a representation in terms of basis functions, called Tchebycheffian B-splines (TB-splines), completely analogous to polynomial B-splines.

Tchebycheffian splines with pieces belonging to null-spaces of constant-coefficient linear differential operators are of particular interest. They grant the freedom of combining polynomials with exponential and trigonometric functions with any number of individual shape parameters [1] and they have been equipped with efficient evaluation and manipulation procedures [4].

TB-splines offer an alternative to standard polynomial B-splines and NURBS in isogeometric methods. It turns out that TB-splines can outperform polynomial B-splines whenever appropriate problem-driven selection strategies for the underlying Tchebycheff spaces are applied [2,3].

In this talk we focus on structure preserving discretizations and we present the construction of discrete differential forms with TB-splines.

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Adaptive Isogeometric Analysis for Volumetric Phase-Field Simulations with Application to Brittle Fracture: Recent Advances and Challenges

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joint work with Lucas Venta Viñuela (UniPV), Luigi Greco (UniPV), Angelos Mantzaflaris (Inria Sophia-Antipolis), Carlotta Giannelli (UniFi) and Alessandro Reali (UniPV)

Since Isogeometric Analysis enables higher-order modeling of phase-field models, it consequently provides computational advantages for the simulation of fracture [1]. In the context of phase-field fracture, several formulations have been proposed in the recent years, including the AT-1 and AT-2 regularizations to model the crack transition region in the context of higher-order formulations [2, 3]. However, several open questions regarding simulations in higher-dimensions, the potential of mesh adaptivity as well as damage initialization using adaptive splines in 3D remain unanswered.
In this talk, we combine our expertise in adaptive meshing, phase-field modeling, phase-field fracture and the implementation of IGA to study the problem of adaptive isogeometric analysis of brittle fracture using phase-fields in 3D. To this end, we employ Truncated Hierarchical B-splines (THB-splines) [4] to form the basis for adaptivity, together with algorithms for admissible meshing [5], since the hierarchical framework provides easy extension to higher dimensions. Using the admissible refinement strategies for THB-splines, we assess different refinement strategies (e.g., mesh projections, marking strategies) and evaluate their performance for volumetric phase-field modeling. Ultimately, we apply this framework to the simulation of brittle fracture in volumetric domains, based on the models presented in [2, 3], where the construction of an initial damage field significantly influences the global structural behavior.

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Diffusing Motion Artifacts for unsupervised correction in brain MRI images

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joint work with Matteo Santacesaria, Vito Paolo Pastore

Motion artifacts are a longstanding obstacle in MRI, degrading image quality and leading to diagnostic uncertainty or costly re-scans. Despite the promise of deep learning solutions, most current approaches rely on supervised training, which demands paired motion-free and motion-corrupted images—a type of data that is extremely rare in clinical practice. This scarcity presents a major roadblock for applying these methods broadly, particularly in routine hospital workflows where patient motion is unpredictable and acquisition parameters vary. In this talk, I'll introduce a framework designed specifically to overcome this data bottleneck. Instead of requiring matched image pairs or k-space access, it uses diffusion models to simulate realistic motion artifacts on clean images, producing synthetic pairs that can then be used to train correction models. This unsupervised pipeline sidesteps the need for controlled acquisition experiments and is compatible with a wide range of clinical scans and hardware setups. I'll present key findings from our evaluation across datasets and acquisition planes, and discuss why solving the data availability challenge is crucial for making AI-based motion correction a practical reality in MRI.

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Non-stationary signal decomposition via deep learning techniques, the IRCNN algorithm

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Real-life signals are non-stationary, and their decomposition is still considered one of the most challenging problems in data processing. Inspired by the successful applications of deep learning in fields like image processing and natural language processing, and given the lack in the literature of works in which deep learning techniques are used directly to decompose non-stationary signals into simple oscillatory components, we use the convolutional neural network, residual structure, and nonlinear activation function to compute in an innovative way the local average of the signal and study a new non-stationary signal decomposition method under the framework of deep learning.

In this talk, we discuss the training process of the proposed model and study the convergence analysis of the learning algorithm. We present a few experiments to show the performance of the proposed model from two points of view: the calculation of the local average and the signal decomposition. Furthermore, we talk about the mode mixing, noise interference, and orthogonality properties of the decomposed components produced by the proposed method.

All results show that the proposed model allows for better handling of boundary and mode mixing effects, and orthogonality of the decomposed components than existing methods, and it proves to be robust to noise perturbation. We conclude the talk with open problems and future research directions.

Mathematical Perspectives on the Explainability of Artificial Intelligence

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Scientific Machine Learning (SciML) has transformed computational science by unifying model-based and data-driven paradigms. A prominent example is Physics-Informed Neural Networks (PINNs), which have emerged as a powerful deep learning framework for solving complex, nonlinear partial differential equations (PDEs) across a wide range of scientific domains.

This presentation explores the theme of explainable SciML, emphasizing key dimensions such as the development of interpretable learning architectures, the integration of domain-specific knowledge, and the reinforcement of computational reliability. Particular attention will be given to methods that promote transparency in neural network models, highlighting approaches that connect theoretical foundations with real-world scientific applications

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Reconstructing high resolution MR images employing MLS-VSK method

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joint work with Isabella Cama, Francesco Marchetti, Emma Perracchione, Cristina Campi

Magnetic Resonance Imaging is a non-invasive imaging technique widely used in medicine to visualize anatomical structures. Its properties make it a leading tool for diagnostic and staging investigations, both in oncological and neurodegenerative diseases. Our main purpose is to address the challenge of low-resolution MR images by reconstructing high-resolution images from low-resolution inputs. This is crucial to improve anatomical detail and diagnostic precision without increasing scan time or patient discomfort. In this direction, we propose a new approximation scheme based on the Moving Least Squares (MLS) method combined with Variably Scaled Kernels (VSK) play the role of the weight function in the weighted L_2 inner product. The MLS scheme enjoys the local approximation based on low-degree polynomials, giving us the advantage of low computational cost compared to other approximation methods (see Chap. 3&4 in [1]). On the other hand, VSK [2] as weight functions enable us to augment some prior information into the approximant, leading to higher precision. In conclusion, our results show that the sugested scheme outperforms the conventional MLS and some other approximation schemes such as partition of Unity.

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To what extent do Convolutional Neural Networks incorporate the wavelet formalism?

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joint work with Vittoria Bruni, Silvia Marconi

This work investigates the theoretical parallelism between the multiscale representation of Continuous Wavelet Transform and the convolutional operators component of CNNs learnt during training. This preliminary study is oriented to the classification of two simple signal categories under noisy conditions. A formal description about how these convolutional operators in each layer can be approximated by a parametric model is then provided. The main result is that the Fourier Transform of the learnt convolutional operators in the frequency-scale domain can be described by a multiscale model. Looking at this new formalism, it is possible to prove that there are deep similarities with the classical multiscale representation of the Wavelet Transform. In particular, it is possible to show that CNNs layers can be linked by a suitable partial differential equation. This study may then help to better understand CNNs structure as well as for improving its performance.

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Geometric methods for analyzing macro-, meso- and micro-structure of textile surfaces

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joint work with Martin Huska, Serena Morigi

The growing demand for virtual representations in textile engineering and design has increased the reliance on digital models of textile surfaces. Understanding the intricate hierarchical structure of textiles is crucial for a multitude of applications.

We study two numerical methods for the multi-scale decomposition of textile surfaces, across their macro-, meso-, and micro-structures. The macro-structure encompasses the overall shape and large-scale features, such as wrinkles and folds. The meso-structure pertains to the arrangement of fibers or yarn woven together to form a fabric (pattern fabric textures); while microstructure refers to the fine details of the warp and weft of fabrics. The first method relies on a curvature-driven differential evolution model leading to a geometric flow blending mean-curvature smoothing with a fidelity term, ensuring that the output surface will retain hierarchical structure and details, [1].

The second approach formulates the decomposition as an energy minimization that balances fidelity to the original surface against a nonconvex sparsity penalty on normal variations, [2]. A tailored Alternating Direction Method of Multipliers solver updates positions and normals field on the surface in a coupled fashion. Multi-scale textile decomposition examples illustrate the limits and advantages of both approaches.

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Noisy Data Approximation Using Pythagorean-Hodograph B-Spline Curves

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joint work with Thierry Bay, Lucia Romani, Laura Saini

The construction of splines for approximating noisy datasets is a classical problem in approximation theory and geometric modeling. Nevertheless this topic has been scarcely explored in the context of Pythagorean Hodograph (PH) splines. In this talk, we begin by recalling a recent method for automatically determining the number and placement of knots in regression spline models [1]. We then show how this idea, with appropriate modifications, can be extended to construct regression splines with multiple knots and optimally placed breakpoints. Building on this framework, we introduce a new approach for constructing planar PH B-spline curves of arbitrary odd degree and knot multiplicity that effectively approximate noisy data. In particular, for a given degree $2n + 1$, a PH spline features C^n continuity and knots of multiplicity $n + 1$ [2]. The construction of the PH spline follows a two-step procedure: first, the nodal partition is determined; then, the spline coefficients are obtained by solving an optimization problem subject to the recently introduced Pythagorean Hodograph constraints [3]. The effectiveness of the proposed method is demonstrated through a series of numerical experiments, highlighting its potential in practical approximation tasks.

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Interpolation and quasi-interpolation by CCC–splines

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Interpolation using Canonical Complete Chebyshev (CCC)–splines, or at least using Extended Complete Chebyshev (ECC)–splines, is well known and has been employed for some time. However, the error bound for such approximations appears to be missing. In this work, we present a derivation of that bound for CCC–splines. In addition to interpolation, we introduce two types of quasi-interpolation, as generalizations of the approximation by CCC–Schoenberg operators. These approximations can be chosen to have an order less than or equal to the order of the associated CCC–spline space. Those of maximal order, when compared to interpolants, exhibit even more desirable behavior.

Fast subdivision of Bézier curves

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joint work with Paweł Woźny

It is well-known that a Bézier curve of degree n can be subdivided at $t \in (0, 1)$ into two segments using the famous de Casteljau algorithm in $O(n^2)$ time. An interesting question arises: can this problem be solved more efficiently? In this talk, we show that it is possible to do this in $O(n \log n)$ time.

Experiments show that the direct application of the new method performs well only for small values of n, as the algorithm is numerically unstable. However, a slightly modified version — which still has $O(n \log n)$ complexity — offers high numerical quality.

Moreover, the new method has a nice property. If a Bézier curve is extended by an additional control point, the subdivision can be updated in $O(1)$ time.

The barycentric form of rational Bézier curves

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joint work with Andriamahenina Ramanantoanina

Rational Bézier curves are indispensable for CAGD due to their numerous favourable properties and the fact that they provide the user with intuitive tools for editing their shape. In this talk, we explore the barycentric form of rational Bézier curves and the additional editing possibilities that it offers.

Quaternion Curves for Pose Control

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Unit quaternions are a convenient method to describe *poses* in 3D space as they are equivalent to the unit sphere. In CNC machine control as well as in the automatic generation of videos, interpolation between or approximation of a discrete set of unit quaternions is needed, but the linearity of standard spline or Bézier curves yields non-unit quaternions which need to be renormalized.

The talk presents a simple trick for the efficient generation of unit quaternion valued curves and some applications.

Optimal uniform parabolic approximation

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joint work with Emil Žagar

In this talk, we consider the problem of finding the best uniform approximant of a planar parametric curve by a parametric polynomial of a fixed degree. So far, only the circular arc approximation, where the error function is simple enough, has been well-studied in the literature. However, the general case is much more difficult than the well-known functional case. The focus will be on parabolic approximation.

Discrete nonlinear reconstructions of functions with discontinuities using quasi-interpolating splines of any degree $d \in \mathbb{N}$.

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joint work with Sara Remogna

We introduce a novel nonlinear reconstruction framework for functions exhibiting discontinuities, employing quasi-interpolating splines of arbitrary degree $d \in \mathbb{N}$. Our approach leverages adaptive approximation techniques and multiresolution analysis to achieve high-order accuracy in smooth regions while effectively capturing discontinuities. The method integrates subdivision schemes and nonlinear weighting strategies to enhance approximation quality. Numerical experiments demonstrate the efficacy of our approach in various scenarios, highlighting its potential for applications in computational mathematics and engineering.

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Smooth surface finishing for 5-axis flank CNC machining of free-form geometries using custom-shaped tools

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joint work with Kanika Rajain, Michal Bizzarri

Computer numerically controlled (CNC) machining is the leading subtractive manufacturing technology and even though it is in use since decades, it is far from being fully solved and still offers a rich source of challenging problems in geometric modeling. Flank, aka peripheral, machining is the finishing stage of the machining process where the tool touches tangentially a to-be-machined surface. The cutting tools used in this stage are typically conical or cylindrical, however, the majority of industrial benchmark geometries like blades or impellers are doubly curved which raises a question whether one shall not use curved tools for manufacturing of curved surfaces, instead of flat ones.

In this talk, we consider generally curved, custom-shaped, cutting tools, whose shape is a design parameter computed by the proposed optimization-based framework to adapt their motions globally to the input free-form surface, supporting a feature of G^1 connection across the neighboring paths. I demonstrate our algorithm on synthetic free-form surfaces as well as on industrial benchmark datasets, showing that optimizing the shape of the tool offers more flexibility to produce G^1 connections between neighboring strips and outperforms conical tools both in terms of the approximation error and the smoothness.

On the localization of the multinode Shepard interpolation formula

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joint work with D. Barrera, F. Di Tommaso, M.J. Ibáñez, F. Larosa, F. Nudo, J.F. Reinoso

The multinode Shepard method is an extension of inverse distance weighting, developed as a generalization of the triangular Shepard method to further improve interpolation accuracy in situations where the classic Shepard method results are limited. In particular, it considers multiple nodes for local interpolation, offering greater flexibility and improved accuracy in estimates. The multinode Shepard interpolation has been proven to be an effective method for reconstructing surfaces from real-world data [1, 2]. In this talk, we present a localized version of the multinode Shepard method and explore its application to the reconstruction of surfaces from digital elevation model (DEM) data.

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Quadrature rules for smooth multivariate splines

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joint work with Carla Manni & Hendrik Speleers (University of Rome Tor Vergata)

Quadrature rules represent a fundamental aspect in several numerical applications and, in particular, in Isogeometric Analysis (IgA) [1]. Using quadrature rules designed for polynomials on simplices as element-wise quadrature rules for smooth splines is a feasible approach. However, this strategy might nullify, or at least significantly reduce, the advantage of smooth splines in IgA because the intrinsic smoothness of the spaces is ignored, resulting in a too high computational cost.

In this talk, we identify multivariate polynomial quadrature rules on simplices that remain exact for sufficiently smooth spline spaces sharing the same degree on the macro-element splits, such as Clough-Tocher, Powell-Sabin [2], Alfeld, Farin-Worsey and Worsey-Piper.

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Enrichment strategy for the standard triangular and simplicial linear finite element

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Low-order elements are widely used and preferred for finite element analysis, specifically the three-node triangular and four-node tetrahedral elements, both based on linear polynomials in barycentric coordinates. They are known, however, to underperform when nearly incompressible materials are involved. The problem may be circumvented by the use of higher degree polynomial elements, but their application becomes both more complex and computationally expensive. For this reason, non-polynomial enriched finite element methods have been proposed for solving engineering problems. In this work, we present a general strategy for enriching the standard linear triangular and simplicial finite element using non-polynomial functions. A key role is played by a characterization result, expressed via the non-vanishing of a certain determinant, which provides necessary and sufficient conditions on the enrichment functions and functionals that guarantee the existence of families of such enriched elements.

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Quasi-Conformal Surface Parameterization via Deep Learning

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joint work with Sofia Imperatore, Carlotta Giannelli

We present a novel approach to surface reconstruction from point clouds that leverages quasi-conformal mapping and deep learning to achieve robust and flexible parameterization. While quasi-conformal theory is well-established in the discrete domain [1], its application to point cloud parameterization learning [2, 3] remains limited. To bridge this gap, we propose a deep learning framework that learns quasi-conformal parameterizations directly from point clouds. We devise a custom loss function that incorporates angular distortion to enforce quasi-conformality, as well as area preservation and boundary conditions. Building on this mapping, we employ advanced spline surface fitting schemes to achieve accurate and efficient surface reconstruction. This approach moves beyond the traditional paradigm of conformal mapping, highlighting the potential of integrating quasi-conformal theory with deep learning.

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Contributed talks

An End-to-End Pipeline for Bézier-Compatible 3D Printing from NURBS Geometry

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Modern 3D printing workflows are evolving to accommodate more sophisticated geometric representations, moving beyond linear approximations to enable smoother, more efficient fabrication. This work considers an end-to-end pipeline that translates Non-Uniform Rational B-Splines (NURBS) into 3D printed objects via a structured sequence of transformations, e.g. NURBS to B-splines, B-splines to Bézier curves, and Bézier curves to G-code commands. Emphasis is placed on the conversion from B-splines to Bézier segments, a critical step that enables compatibility with G-code commands—native support for Bézier curves in many modern 3D printing systems. By segmenting B-splines at knot intervals and applying precise control point conversions, this method ensures continuity and fidelity across the curve transformation stages. Leveraging native Bézier-compatible G-code allows for smoother toolpaths, improved surface finishes compared to traditional linear (G1) approximations, and possibly other optimizations, such as improved toolpaths. Furthermore, by enabling the direct use of NURBS-derived data in the printing process, this workflow minimizes geometric loss and preserves design intent, offering significant advantages in precision manufacturing, especially for freeform and high-curvature surfaces. We propose a pipeline where advanced curve representations can be utilized in additive manufacturing and investigate the performance with respect to enhancing both quality and efficiency.

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[2] J. M. Chacón, J. Sánchez-Reyes and J. Vallejo, *G-code generation in a NURBS Workflow for precise additive manufacturing*, Rap. Prot. Jour. **28/11: 65-76** (2022)

[3] Michael J. Borden, Michael A. Scott, John A. Evans and Thomas J. R. Hughes, *Isogeometric finite element data structures based on Bézier extraction of NURBS*, Int. J. Numer. Meth. Engng; **87:15–47** (2011)

Expo-Rational B-spline volumes

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joint work with Rune Dalmo, Aleksander Pedersen

Expo-Rational B-splines, ERBS, has so far been utilized mainly for curves and surfaces. A tensor product volume ERBS will like its surface and curve equivalents have a strong localization property. As with local curves and surfaces, local volumes will also give us some control over the variation within the parameterization. With local volumes on Bezier form, representing derivatives in a knot, we suggest using this for adaptive grids and meshing. By changing the dimension of local volumes, we can introduce singularities in the resulting ERBS volume that is interesting in the context of grid refinement. The blending of information in the volume is strictly local between the knots and combined with a data structure suitable for visualization and computation this opens some possibilities for novel applications in fused deposition modeling, and finite elements.

On Autoencoders and Sparse Representation of Patterns

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We discuss the use of deep neural networks autoencoders and their application for discovering sparse representations. A particular attention will be given to classifying specific patterns in portions of an image.

Neural Network design for the selection of the optimal HP-Spline frequency parameter

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joint work with V. Bruni, R. Campagna, C. Conti, D. Vitulano

We explore the use of artificial neural networks for the data-driven selection of the frequency parameter in hyperbolic polynomial penalized splines (HP-splines). This parameter, which defines the underlying spline space, is crucial to adapt the model to exponential patterns in the data, such as those encountered in signal processing. We investigate how neural network architectures can help estimate this parameter effectively, taking into account the influence of noise and data distribution. The approach aims to extend recent findings on the approximation complexity of neural network, with the goal of defining a mathematical error model. Preliminary results and numerical experiments provide encouraging evidence.

[1] R. Campagna, C. Conti, *Penalized hyperbolic-polynomial splines*, Applied Mathematics Letters, vol. 118 (2021)

[2] R. Campagna, C. Conti, S. Cuomo, *A linear algebra approach to HP-splines frequency parameter selection*, Appl. Math. Comput.,vol. 458 (2023)

[3] V. Bruni, R. Campagna, D. Vitulano, *Multicomponent signals interference detection exploiting HP-splines frequency parameter*, Applied Numerical Mathematics, vol. 209 (2025)

[4] Yarotsky, Dmitry. "Error bounds for approximations with deep ReLU networks." Neural networks 94 (2017)

[5] C. C. Aggarwal, *Neural Networks and Deep Learning*, Springer (2025)

Quadrature rules based on second-order Bernstein-like operators

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joint work with Giorgia Bronzo, Paola Lamberti

In this work we propose new quadrature rules based on a generalization [1] of a class of second order Bernstein-like operators, presented in [2].

Such a generalization introduces a parameter h, that can be used to increase the degree of precision of the above rules.

Several results confirming the numerical integration performances are shown.

1 B. Azzarone, P. Lamberti, *On a wider class of second-order Bernstein-like operators*, submitted

2 H. Khosravian-Arab, M. Dehghan , M. R. Eslahchi, *A new approach to improve the order of approximation of the Bernstein operators: theory and applications*, Numer. Algor. **77**, 111-150 (2018)

Hermite-type sampling operator

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This talk concerns the topic of approximation theory and it is based on [1].
The generalized sampling operator is able to approximate bounded continuous functions and it is modeled on the sampling expansion for band-limited functions given by the Whittaker-Kotel'nikov-Shannon theorem. During the decades, some variations of this classical theorem have been proposed. One of them (dating back to Jagerman and Fogel and, in a more general form, to Linden and Abramson) takes into consideration also the derivative samples for the reconstruction of band-limited functions, with a consequent benefit of a larger sampling rate compared to the Whittaker-Kotel'nikov-Shannon theorem. Motivated by this new reconstruction, we modify the generalized sampling operator including the samplings of derivatives up to a generic order to approximate non necessarily band-limited functions. The definition of the new operator is

$$(G_{n,w}f)(x) = \sum_{k\in\mathbb{Z}} \left(\sum_{j=0}^{n} \frac{1}{j!} f^{(j)}\left(\frac{k}{w}\right) \left(x - \frac{k}{w}\right)^{j} \right) \chi(wx - k), \qquad x \in \mathbb{R},$$

(where χ is, as usually, a kernel) and we call it an *Hermite-type sampling operator* of order $n \in \mathbb{N}^+$. One of its main features is the faster order of approximation. Besides the convergence and its rate, we discuss well-posedness, regularity, simultaneous approximation and a Voronovskaya-type formula.

[1] R. Corso, Generalized sampling operators with derivative samples. J. Math. Anal. Appl., 547(1), 129369, (2025).

Wavelet-based non-parametric estimation of self-similar fractals

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joint work with Lubomir T. Dechevsky

The Peetre K-functional between Besov spaces can be used as a tool for measuring the regularity of curves and the quality of fit by wavelet estimation. For this purpose, it is necessary to use equivalent quasi-norms in the Besov spaces based on weighted sequences of wavelet coefficients. This approach leads to optimal filtering methods based on non-linear shrinkage of noisy wavelet coefficients.

Our work builds on [1] as we consider the self-similar fractal estimator outlined there.

We study the advanced performance of the James-Stein estimator in the case when the noisy data refer to a non-parametric self-similar fractal. For this purpose, we employ a hard-threshold wavelet estimator based on weighted decreasing re-arrangement of the wavelet coefficients. Results obtained via this approach are compared with those obtained via the standard James-Stein soft thresholding estimator.

Our approach is of interest for various engineering applications, e.g., pattern recognition, data compression, and image processing.

[1] Dechevsky, L.T., Ramsay, J.O., Penev, S.I.: Penalized wavelet estimation with Besov regularity constraints. Math. Balkanica 13(3–4), 257–376 (1999).

A decoupled meshless Nyström scheme for 2D Fredholm integral equations of the 2$^{\text{nd}}$ kind with smooth kernels

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joint work with Alessandra Sestini

Discretizing the Fredholm integral equation of the second kind

$$\lambda u(\mathbf{x}) - \int_{\Omega} k(\mathbf{x}, \mathbf{y}) u(\mathbf{y}) \, d\mathbf{y} = f(\mathbf{x}) \tag{1}$$

with kernel k using collocation or Galerkin's method leads to a linear system whose size depends on the dimension of the chosen trial space, but not on the quadrature rule used to integrate over the bounded multivariate domain $\Omega \subset \mathbb{R}^d$. This contrasts with the *classical* Nyström method, where the degrees of freedom in the numerical solution are identified with the set of nodes Y of a global quadrature formula over Ω. This proves inefficient if the kernel k varies much more rapidly than the solution u: in such cases, collocation and Galerkin's methods can profitably combine a coarse trial space with arbitrarily fine quadrature, whereas the classical Nyström method must solve an unnecessarily large squared system with $|Y|$ unknowns.

In this talk, we introduce a high-order *decoupled* variant of the Nyström method, where the node set X for approximating u and the node set Y for integrating $k(\bar{\mathbf{x}}, \mathbf{y})u(\mathbf{y})$ over Ω (with $\bar{\mathbf{x}} \in X$) can be chosen independently. The case $|X| \ll |Y|$ is of practical interest, because the decoupled scheme produces a squared system with only $|X|$ unknowns. We prove, using standard tools from functional analysis, that arbitrarily high orders of convergence can be achieved under natural assumptions.

Theoretical arguments and numerical experiments illustrate the computational advantages of the decoupled scheme, especially for rapidly-varying kernels. This work focuses on smooth kernels and 2D domains, paving the way for further generalizations. Numerical experiments are performed in a meshless setting, with X and Y being sets of scattered nodes over $\overline{\Omega}$. Quadrature employs the recently developed high-order moment-free approach described in the PhD thesis [1]. Applications to population dynamics are presented.

[1] B. Degli Esposti, *Domain discretization and moment-free quadrature for meshless methods*, flore.unifi.it/handle/2158/1417355, (2025)

Multiresolution techniques in Large Scale Optimization problems

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joint work with Sergio Lopez-Ureña, Anna Martinez-Gavara

The cost of 'off-the shelve', general purpose optimization techniques may become prohibitive when the number of degrees of freedom in the problem is very large. By embedding the original optimization problem within *Harten's Multiresolution Framework* [1], we are able to compute the desired solution after computing a finite sequence of *suboptimal solutions* of auxiliary optimization problems that involve an increasing number of degrees of freedom. We show that this *multilevel approach* provides a computationally efficient strategy, which can greatly speed up the performance of a prescribed optimization technique while still allowing the end user to treat both the optimizer and the objective funcion of the problem as black boxes throughout the optimization process.

This strategy stems from a practical problem described in [2]

[1] A. Harten, *Multiresolution representation of data: A general framework* SIAM J. Num. An. 33 (1996) 1205-1256

[2] R. Donat, S. Lopez-Ureña, M. Menec, *A novel multi-scale strategy for multi-parametric optimization* European Consortium for Mathematics in Industry, (2016) 593-600

Curves approximation based on inverse multiquadric radial basis functions

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joint work with Giulia De Santis, Josephin Giacomini, Pierluigi Maponi, Alessia Perticarini

We consider the problem of fitting a set of points in the plane to a smooth curve. For solving this problem, we construct, by using inverse multiquadric radial basis functions, an approximating curve starting from the given points. The quality of the obtained approximation is also studied, with particular attention to its dependance on the choice of the shape parameter.

The obtained results are compared with the approximation obtained by splines, with both synthetic and real data.

Mixed Derivatives Total Variation

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joint work with Michaël Unser

The formulation of norms on continuous-domain Banach spaces, with an exact pixel-based discretization, is beneficial for the resolution of inverse problems (IPs). In this paper, we study a new regularization that is the convex combination of a TV term and the $\mathcal{M}(\mathbb{R}^2)$ norm of mixed derivatives. We show that the extreme points of the unit ball are indicators on polygons with edges oriented in the x_1 or the x_2 dimension. We apply this result to build a new regularization for IPs, that is exactly discretized by tensor products of B-splines of order 1 or, equivalently, pixels. We exactly discretize the loss of the denoising problem on its canonical pixel basis, and show that it has a unique solution, which also is a solution of the underlying continuous-domain IP.

A direct slicing approach for additive manufacturing

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joint work with Rune Dalmo, Børre Bang

In a typical workflow for additive manufacturing (AM), the CAD model is triangulated and converted into an STL file format. A slicing process is then performed on the mesh to obtain the layers needed for the AM process. Due to the triangulation, inaccuracies may occur, e.g. conversion error, lack of data, inaccurate data representation and approximation. Our approach is to perform slicing of the NURBS representation directly in the CAD software to avoid conversion or exchange of the model between software packages. We investigate how a NURBS based representation without the meshing step performs with respect to data sampling and approximation error.

Nearly interpolating monotonicity preserving C^1 subdivision scheme with the third-order accuracy

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joint work with Jungho Yoon

Among the features of a subdivision scheme, monotonicity preservation is particularly important for practical applications. This study proposes a nearly interpolant subdivision scheme that preserves monotonicity of the given initial data. Unlike most existing algorithms, which are often restricted to *strictly* monotone data, the proposed scheme works regardless of strictly or non-strictly monotone. To this end, we introduce a modified *minmod* technique, that acts a limiting procedure to suppress spurious oscillations. The resulting scheme achieves third-order accuracy and endures C^1 smoothness. Some experimental results are presented to illustrate the accuracy, smoothness and monotonicity preserving capability of the proposed scheme.

Isogeometric collocation with smooth mixed degree splines over planar multi-patch domains

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joint work with Mario Kapl, Vito Vitrih

We present a novel isogeometric collocation method for solving the Poisson's and the biharmonic equation over planar bilinearly parameterized multi-patch geometries. The proposed approach relies on the use of a modified construction of the C^s-smooth mixed degree isogeometric spline space [1] for $s = 2$ and $s = 4$ in case of the Poisson's and the biharmonic equation, respectively. The adapted spline space possesses the minimal possible degree $p = s + 1$ everywhere on the multi-patch domain except in a small neighborhood of the inner edges and of the vertices of patch valency greater than one where a degree $p = 2s + 1$ is required. This allows to solve the PDEs with a much lower number of degrees of freedom compared to employing the C^s-smooth spline space [2] with the same high degree $p = 2s + 1$ everywhere. To perform isogeometric collocation with the smooth mixed degree spline functions, we introduce and study two different sets of collocation points, namely first a generalization of the standard Greville points to the set of mixed degree Greville points and second the so-called mixed degree superconvergent points. The collocation method is further extended to the class of bilinear-like G^s multi-patch parameterizations [3], which enables the modeling of multi-patch domains with curved boundaries, and is finally tested on the basis of several numerical examples.

1. M. Kapl, A. Kosmač, V. Vitrih: *A C^s-smooth mixed degree and regularity isogeometric spline space over planar multi-patch domains.* https://arxiv.org/abs/2407.17046, 2024.

2. M. Kapl, V. Vitrih: *C^s-smooth isogeometric spline spaces over planar multi-patch parameterizations.* Advances in Computational Mathematics, 47:47, 2021

3. M. Kapl, V. Vitrih: *Dimension and basis construction for C^2-smooth isogeometric spline spaces over bilinear-like G2 two-patch parameterizations.* J. Comput. Appl. Math., 335:289–311, 2018.

Trivariate blending type spline constructions in isogeometric applications

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Trivariate spline volumes have attracted considerable interest in recent years due to their applications in isogeometric analysis. This talk introduces an alternative representation of analysis-suitable volume parameterization based on tensor product blending type spline constructions. These constructions blend local geometries using a combination of expo-rational local functions and overlapping Bernstein polynomials as the spline basis.

Several methods for generating solid models using trivariate blending type spline constructions are presented, including manual design and mapping from CAD geometries. The resulting spline volumes are employed as computational domains for solving partial differential equations and can be directly used in isogeometric frameworks.

This construction offers notable flexibility across a range of applications. Its key advantage lies in preserving its structural consistency throughout all stages of the simulation process: modeling, computations, and optimization.

The benefits of the proposed construction are illustrated through several examples, highlighting its performance compared to traditional spline methods.

Geometric parametrization of C^1 Hermite interpolants by spatial quintic Pythagorean hodograph curves based on extremality

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joint work with Chang Yong Han

The interpolation of C^1 Hermite data by spatial quintic Pythagorean hodograph (PH) curves results in a two-parameter family of solutions, as shown in [1]. To explicitly represent all such interpolants, various parametrization methods can be employed. Šír and Jüttler [2] proposed one such parametrization, which possesses several desirable geometric properties, including parameter invariance under rotations.

In this study, we introduce a new geometric parametrization of these interpolants, based on invariants associated with the extremality of PH curves [3, 4]. These invariants can be computed from intrinsic properties of the corresponding quadratic preimages, enabling the parameters of a given interpolant to be determined directly—without the need to transform the interpolant into a standard position.

The proposed parametrization exhibits several geometric advantages, including invariance under orthogonal transformations and straightforward identification of planar interpolants.

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[2] Z. Šír, B. Jüttler, Spatial Pythagorean hodograph quintics and the approximation of pipe surfaces, in: R. Martin, H. Bez, M. Sabin (Eds.), Mathematics of Surfaces XI, Springer Berlin Heidelberg, Berlin, Heidelberg, 2005, pp. 364–380.

[3] C. Y. Han, H. P. Moon, S.-H. Kwon, A new selection scheme for spatial Pythagorean hodograph quintic Hermite interpolants, Computer Aided Geometric Design 78 (2020) 101827.

[4] C. Y. Han, S.-H. Kwon, Motion interpolation with Euler–Rodrigues frames on extremal Pythagorean-hodograph curves, Mathematics and Computers in Simulation 234 (2025) 325–341.

A Bivariate Spline Construction of Orthonormal Polynomials over Polygonal Domains and Its Applications to Quadratures

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I will explain how to use bivariate splines for constructing orthogonal polynomials over any polygonal domain in $\mathbb{R}^2$. One of the motivations of the study is to compute numerical integration of any continuous functions over any polygon. I shall first review Gaussian quadrature and one point quadrature and point out their extensions. Then I will explain that the one point quadrature is exact for more than half of the functions in $C([-1,1])$. This motivates us to compute orthogonal polynomials in the multivariate settings. Two computational algorithms will be presented: one is to construct orthonormal polynomials of degree d over a polygon and one is to construct orthonormal polynomials of degree $d+1$ in the orthogonal complement of the polynomial space P_d in P_{d+1}. In particular, I will use my MATLAB implementation of bivariate splines to realize these algorithms. Many numerical examples of orthogonal polynomials of various degree $d = 1, \cdots, 5$ will be given. Then I will use the orthogonal polynomials to construct numerical quadrature in the 2D setting. Several interesting and useful examples will be given for triangle and other n–side polygonal domains with $n \geq 4$.

[1] G. Awanou, M. -J. Lai, P. Wenston, The multivariate spline method for numerical solution of partial differential equations, in Wavelets and Splines, Nashboro Press, Brentwood, 2006, pp. 24–74.

[2] M. -J. Lai and L. L. Schumaker, Spline Functions over Triangulations, Cambridge University Press, 2007.

[3] Y. Xu, Orthogonal polynomials of several variables, Multivariable special functions, Cambridge University Press, 2021, pp. 19–78.

Subdivision for Splines on Vertex Stars

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joint work with Bert Jüttler

We follow the free-form spline construction introduced by Prautzsch [1], which provides a valuable tool for constructing smooth surfaces by extending tensor product splines to allow for extraordinary vertices.

Our goal is to extend this approach to isogeometric analysis by constructing a generating system of functions that satisfy certain properties. To achieve this, we start our construction with splines defined by a vertex star configuration, where the valence ($\neq 4$) is determined by the extraordinary vertex. Using the available degrees of freedom, we strategically choose Greville points to ensure full symmetry. In addition, the coefficients are chosen in such a way that the resulting system is a partition of unity.

In our talk, we will discuss further constraints and methods for the optimal selection of these functions. In particular, we focus on the handling of linear dependencies and introduce a subdivision scheme.

[1] H. Prautzsch: Freeform splines. Computer Aided Geometric Design, Vol. 4 Issue 3, pp.201-206, 1997

Reconstruction of discontinuous functions via integral data and multinode Shepard functions

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joint work with F. Dell'Accio, F. Nudo, N. Siar

Histopolation, also known as interpolation over segments, is a mathematical method used to approximate a function f on a given interval $I = [a, b]$ by leveraging integral information over specific subintervals of I. Unlike traditional polynomial interpolation, which relies on pointwise function evaluations, histopolation reconstructs a function using integral data. However, much like classical interpolation, it is affected by the Runge phenomenon when employing a uniform grid with numerous equispaced nodes, and by the Gibbs phenomenon when approximating discontinuous functions. To overcome these challenges, quasi-histopolation has been introduced as a more flexible alternative that does not strictly enforce interpolation through all data points. This added flexibility reduces the tendency for oscillatory behavior, particularly when employing rational approximation techniques. In this work, we propose a novel C^∞ rational quasi-histopolation operator designed for bounded, integrable functions. Our approach effectively mitigates both the Runge and Gibbs phenomena by blending multinode Shepard functions with local histopolation polynomials based on a limited number of nodes. Numerical experiments confirm the accuracy and robustness of the proposed method.

[1] Dell'Accio, F., Di Tommaso, F., Hormann, K. (2018). Reconstruction of a function from Hermite–Birkhoff data. *Applied Mathematics and Computation*, 318, 51–69.

[2] Bruni Bruno, L., Erb, W. (2024). Polynomial interpolation of function averages on interval segments. *SIAM Journal on Numerical Analysis*, 62(4), 1759–1781.

Shape preserving approximation by trigonometric polynomials

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joint work with I.&V. Shevchuk

We are concerned with approximating a continuous 2π-periodic function f, that changes its monotonicity finitely many times in each period, by trigonometric polynomials that exactly follow the monotonicity of the function, that is, comonotone with f. Since a nonconstant trigonometric polynomial may have only an even number of extrema in each period, we must limit ourselves to functions f that have $2s$, $s \geq 1$, extrema in a period a collection we denote by $Y_s : -\pi < y_1 < \cdots y_{2s} \leq \pi$.

Let

$$E_n(f) := \inf \|f - T_n\|, \quad n \geq 1,$$

where the infimum is taken over all trigonometric polynomials of degree $< n$, denote the (unconstrained) nth degree of approximation of f.

If f has Y_s as its extrema in $(-\pi, \pi]$, let

$$E_n^{(1)}(f, Y_s) = \inf_{T_n \text{ comonotone with } f} \|f - T_n\|,$$

denote the nth degree of comonotone approximation of f.

Clearly $E_n(f) \leq E_n^{(1)}(f, Y_s)$, but the opposite inequality (obviously, even with a constant multiple) is, in general, invalid.

We will show that if $r \geq 2s - 1$ and

$$(*) \qquad E_n(f) \leq n^{-r}, \quad n \geq 1,$$

then

$$E_n^{(1)}(f, Y_s) \leq c(r) n^{-r}, \quad n \geq 1.$$

However, if $1 \leq r \leq 2s - 2$, then there is a constant $c(r) > 0$, such that for each $m \geq 3$, there is a collection Y_s and an f with Y_s its extrema in $(-\pi, \pi]$, satisfying (*), but

$$m^r E_m^{(1)}(f, Y_s) \geq c(r) \ln m.$$

[1] D. Leviatan and I. O. Shevchuk, Shape preserving approximation of periodic functions – Conclusion, Constr. Approx. (2005) (to appear)

[2] D. Leviatan, I. Shevchuk and V. Shevchuk, Comparing the degree of constrained and unconstrained trigonometric approximation, J. Approx. Theory (to appear)

Hierarchical Biquadratic Splines with almost C^1 continuity for Isogemetric Analysis simulations

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joint work with Cesare Bracco, Carlotta Giannelli, Thomas Takacs, Deepesh Toshniwal

We introduce a novel class of multipatch hierarchical spline spaces designed for adaptive isogeometric analysis of fourth-order partial differential equations. These spaces extend the concept of almost–C^1 biquadratic splines, as presented in [3], to a hierarchical framework that enables local refinement. The construction ensures C^1 continuity at regular and extraordinary vertices and along regular edges, while allowing C^0 continuity only across edges adjacent to extraordinary vertices. Differently from existing approaches (e.g., [1,2]), no gluing functions are required, leading to a simpler yet robust formulation. Theoretical aspects, such as the nestedness of the hierarchical spaces away from extraordinary vertices and the linear independence of their basis functions, are also addressed. Numerical experiments confirm the accuracy and flexibility of the proposed method.

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[2] C. Bracco, A. Farahat, C. Giannelli, M. Kapl, R. Vázquez, *Adaptive methods with C^1 splines for multi-patch surfaces and shells*, Comput. Methods Appl. Mech. Engrg. 431 (2024).

[3] T. Takacs, D. Toshniwal, *Almost-C^1 splines: Biquadratic splines on unstructured quadrilateral meshes and their application to fourth order problems*, Comput. Methods Appl. Mech. Engrg. 403 (2023).

Approximation methods for high degree polynomial surfaces using Gauss–Legendre basis

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joint work with Jingxuan Li, Soo Hyun Kim, Hans-Peter Schröcker, Hyojeong Park

Tensor product Bézier patches are the most fundamental surface primitives used in Computer Aided Geometric Design (CAGD). Although Bézier patches can be defined and computed regardless of degree, it is rare to use a single piece of high degree Bézier patch to design a surface with complicated shape. For high degree polynomial surfaces, it is difficult to manipulate the shape of surfaces using Bézier control nets, because the value of Bernstein basis decreases and the effect of individual control points gets weaker as the degree increases.

Recently, a new polynomial basis called Gauss–Legendre (GL) polynomials [1] has been developed for the design of high degree curves. We here present some methods of approximating given parametric surfaces using high degree polynomial tensor product patches based on the GL basis. We first determine the corner points or the boundary curves of the patch, then compute the interior control points of the GL patch using the data such as the tangent vectors, the normal vectors, and the twist of the reference surface.

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Spline curves on tensor product surfaces

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joint work with Arne Lakså

We construct spline curves mapped on surfaces and their derivatives using basic differential geometry and compare their properties with some of the results discussed in [2], [3], [4]. We show examples on open and closed curves on different tensorproduct surfaces. The notation follows [1].

Let $U \subset \mathbb{R}^2$ be the domain for a parametric surface $S \subset \mathbb{R}^3$, with parameters u, v. The mapping $S : U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ can be written as $S(u,v) = (u,\ v,\ f(u,v)\ )$. The differential $dS = (\frac{\partial S}{\partial u} \quad \frac{\partial S}{\partial v})$ is a matrix where $\frac{\partial S}{\partial u} = (1,\ 0,\ \frac{\partial f}{\partial u})$ and $\frac{\partial S}{\partial v} = (0,\ 1,\ \frac{\partial f}{\partial v})$. We can construct a mapping $c(t)$ on S for a curve $h(t) \subset U$ by $c(t) = S \circ h(t) = s(u(t), v(t))$. We obtain expressions for the derivatives $c'(t),\ c''(t),\ c'''(t)$ by repeatedly applying the differential dS. These expressions is implemented by modifying existing algorithms.

[1] A.Lakså. *Blending techniques in Curve and Surface constructions.* Geofo, 2022.

[2] C.Mancinelli, E.Puppo. Splines on manifolds: A survey, *Computer Aided Geometric Design*, 112, 2024

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Towards hierarchically refinable spline complexes for adaptive simulations on irregular triangulations

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This presentation suggests a possible avenue towards constructing hierarchically refinable spline spaces on irregular triangular meshes that constitute differential complexes and thus allow for adaptive and yet structure-preserving simulations.

The construction we want to explore has multiple steps. First, we need to identify spaces of refinable splines that can be combined into a differential complex. These spaces then need to be generalized to irregular meshes while maintaining their refinability, for example through subdivision. Finally, the refinability is leveraged to construct hierarchical spaces that still comprise differential complexes with favorable properties. Each step comes with its own set of technical challenges.

The purpose of this presentation is to outline the foundations we have already established in our work, including the construction of hierarchical, structure-preserving $\mathcal{C}^0$ finite element spaces based on subdivision. Starting from there, we consider possible extensions of these finite elements that bring us closer to the construction proposed earlier. Along the way, the presentation will shine a light on known complications and open problems in this context and suggest approaches to overcome them wherever possible.

Image Reconstruction from Undersampled Fourier Data

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In applications such as Magnetic Resonance Imaging (MRI), data are acquired in the Fourier domain and to accelerate imaging, a subsample of the discrete Fourier data is often taken. In order to recover the image, we need to reconstruct the missing Fourier data and then apply the discrete inverse Fourier transform. However, reconstructing missing data in the Fourier Space poses significant challenges. Fourier transformed images exhibit strong oscillations, which means that the Fourier data cannot be not as easily reconstructed. In this talk, we will show why linear interpolation in the Fourier domain is futile and discuss alternative reconstruction methods, as e.g. "Generalized Autocalibrating Partially Parallel Acquisitions" (GRAPPA), a widely adopted approach in MRI reconstruction. Furthermore, we will introduce our new hybrid algorithm, which combines TV minimisation with another method that iteratively updates the data in the *image domain* using local variations.

The Multichannel Blind Deconvolution Problem in Parallel MRI

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One of the biggest innovations in magnetic resonance imaging (MRI) within the last years was the concept of parallel MRI. In this setting, the use of multiple receiver coils allows the reconstruction of high-resolution images from undersampled Fourier data such that the acquisition time can be substantially reduced. Mathematically, the parallel MRI reconstruction problem can be seen as a multi-channel blind deconvolution problem, where the coil sensitivity functions and the magnetization image have to be recovered simultaneously from the acquired data.

In this talk, we will give a short survey on existing reconstruction methods in parallel MRI so far. Extending the recently derived MOCCA algorithm, we propose to employ a new model for the coil sensitivity functions, where the so-called sum-of-squares condition is connected with a small support in k-space. We will show that the resulting algorithm is well applicable for real MRI data and outperforms many other related algorithms based on subspace or interpolation methods.

Recognition of geometric primitives from point clouds for buildings' reconstruction

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joint work with Silvia Biasotti, Bianca Falcidieno

In Computer-Aided Design (CAD), recognizing geometric primitives and feature curves from point clouds (PCs) is a well-researched topic. This recognition is crucial for reconstructing digital models that can be broken down into simple components, allowing for easy manipulation by CAD systems. CAD objects are usually well sampled, producing dense and regular PCs of limited size. These PCs are generally free of large occlusions or excessive noise.

In contrast, PCs representing urban environments are typically much larger and can be significantly impacted by noise and gaps caused by inherent or occasional occlusions. Therefore, it is crucial to develop robust recognition methods that allow for lightweight reconstructions of built structures using simple volumetric or surface primitives (planes, cylinders, cones and spheres), as well as facade openings and accessories through feature curves.

In this work, we present a method based on the Hough transform to identify surface geometric primitives and feature curves that collectively define a building. Our approach starts from a collection of 3D points that represent the shape of a building's exterior. These PCs are generated using 3D laser scanners or other technologies and consist of a combination of terrestrial and aerial data. We begin by identifying a combination of geometric primitives to extract the architectural elements that make up the building, such as the roof and facades. To achieve this, we adapt a previous approach designed for CAD models [1] for use in an urban context. Additionally, we enhance the identification of facade features by using a revised version of the method [2].

The outcome is a compact geometric representation of a building, enhanced with essential semantic information needed to create its digital twin.

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Exploring nonlinear approaches to quasi-interpolation: techniques and analysis

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In this talk we present some innovative quasi-interpolation techniques using nonlinear strategies to manage data discontinuities across one and several dimensions. The main advancement involves applying nonlinear weights to quasi-interpolators of the form,

$$\mathcal{Q}(f) = \sum_{i=1}^{N} L_i(f) a_i(\mathbf{x}), \tag{2}$$

where L_i, a_i, or both form a partition of unity. We adapt data reconstruction methods for multivariate gridded and non-gridded data that present discontinuities. We analyze the numerical properties of these schemes, including smoothness, accuracy near discontinuities, and elimination of the Gibbs phenomenon. Numerical experiments in various dimensions support our theoretical results.

Construction of rational C^1 cubic Powell–Sabin splines

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joint work with Jan Grošelj

In this talk we introduce rational C^1 cubic Powell–Sabin splines [1] and analyse their basic properties. The splines are constructed over a Powell–Sabin refinement of a given triangulation, obtained by splitting each triangle into six smaller triangles. We first provide rational B-spline basis functions that possess three favorable properties; local support, nonnegativity, and a partition of unity. Then we present a procedure for determining the spline control points and weights by using the blossoming operator and discuss the relation of the introduced splines to Bézier surfaces and NURPS splines [2]. We conclude with a few applications and practical examples including the representation of some planar and spatial ruled surfaces.

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Mathematical Modeling and Automated Adjustment of the Focusing Optics in Free-electron Lasers

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joint work with Gerlind Plonka, Klaus Mann, Bernd Schäfer

FLASH is a free-electron laser that produces femto second short pulses of X-ray radiation. To enable precise experimental applications, the beam must be focused using a Kirkpatrick-Baez (KB) mirror system. It consists of two mirrors, which can be bent, rotated and translated, meaning that there are 12 controllable degrees of freedom. Currently, these parameters are manually adjusted before each experiment—a process that is both time-intensive and requires the presence of a human expert. We aim to automate the choice of these mirror settings, depending on the varying properties of the incoming beam and on the experiment's requirements. We present in this talk our model for simulating the propagation through the mirror system as well as methods to solve the resulting optimization problem.

Boundary Integral Methods on Irregular Domains

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Boundary integral methods (BIMs) offer a powerful computational framework for solving partial differential equations. Unlike domain-based methods, BIMs reduce problem dimensionality by formulating solutions in terms of boundary values, naturally incorporating jump conditions at interfaces. This is particularly beneficial in applications such as electrostatics, fluid dynamics, and elasticity, also in cases where boundaries feature geometric singularities, sharp corners, or heterogeneous material interfaces. By focusing computations on the boundary, BIMs can provide high accuracy with reduced computational cost, even in domains where traditional meshing becomes challenging or unstable. As such, BIMs remain a method of choice for high-fidelity modeling in irregular domains across a wide range of engineering and physical sciences.
We focus on the quadrature error associated with a regular quadrature rule for evaluation of a layer potential near the boundary where the integral becomes nearly singular. We present quadrature error estimates for integrals with various types of singularities, using both the trapezoidal and Gauss–Legendre quadrature rules. These estimates are derived for integrals over curves using complex analysis techniques, including contour integrals, residue calculus, and branch cuts. The resulting error estimates are computationally efficient and practically useful. Numerical examples are provided to illustrate the performance of the estimates on boundaries with different levels of regularity.

New approach for constructing edge B-spline-like basis functions for C^1 and C^2 splines over triangulations

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Splines over triangulations provide a flexible and efficient way to approximate bivariate functions and surfaces defined over domains, partitioned by triangles. One of the important challenges is the construction of locally supported non-negative basis functions that form a partition of unity, i.e. B-spline-like basis functions. Due to smoothness conditions that depend on the geometry of the triangles in the triangulation, the basis functions are usually separated into three groups – triangle, vertex and edge group. While the construction of basis splines in the first two groups is well developed, ways to compute the nonnegative edge basis splines of general degree forming the partition of unity are explored only for C^1 continuous splines (see [1]). The drawback of the construction proposed in [1] is its lack of a geometric interpretation. In this talk a novel approach is presented that can be geometrically explained and generalized to splines of a higher smoothness. Due to the desired property of local support, the study can be limited to two triangles that share an edge. However, to be able to combine this construction with the construction of vertex basis functions certain constraints must be taken into account. We describe how the new method generates C^1 and C^2 continuous splines over two triangles forming a convex quadrilateral, discuss the potential extension to higher degrees of smoothness, and explain how to use the results for splines over general triangulations.

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Genuine multi-sided surfaces: an overview and recent progress on curved domain Bézier and B-spline patches

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joint work with Péter Salvi, Tamás Várady

While free-form surfaces in practical CAD systems are currently designed using tensor-product Bézier and B-spline surfaces, there exist application areas, such as curve network design, hole filling and blending where genuine multi-sided surface representations might offer substantial advantages. We first present a high-level overview of the techniques and challenges specific to the field of multi-sided surfaces, following our recent survey [1]. We then focus on recent developments [2] that aim to generalize regular Bézier/B-spline control structures to patches defined over arbitrary (even multiply-connected) subsets of the plane. The construction is based on a parametric variant of the medial axis, and can be adapted to tune the interior of any multi-sided surface.

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Planar PH curves in generalized polynomial spaces of order 4 and 6

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joint work with Lucia Romani, Alberto Rossi

We present a general framework for planar PH curves in generalized polynomial spaces of order 4 and 6, which includes standard polynomials along with trigonometric and exponential ones. First, from a theoretical point of view, we generalize the algebraic and geometric characterizations introduced in [1] to the trigonometric and exponential case. Then, from a more practical point of view, following the ideas proposed in [2] and [3], we study under which conditions it is possible to construct a PH curve given two of the control edges and to modify some of the control points without losing the PH property.

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An Isogeometric Tearing and Interconnecting (IETI) method for solving high order partial differential equations over planar multi-patch geometries

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joint work with Mario Kapl, Aljaž Kosmač

A novel method for solving high-order partial differential equations (PDEs) over planar multi-patch geometries will be presented and demonstrated on two model problems given by the biharmonic and triharmonic equation. The approach is based on the concept of Isogeometric Tearing and Interconnecting (IETI) [1] and allows to couple the numerical solution of the PDE with C^s-smoothness across the interfaces of the multi-patch geometry. The proposed technique relies on the use of a particular class of multi-patch geometries, called bilinear-like G^s multi-patch parameterizations [2], to represent the multi-patch domain. The coupling between the neighboring patches is discussed in detail, in particular to connect the numerical solution with C^1 and C^2-smoothness, which is required for solving the biharmonic and triharmonic equation, respectively. Several numerical examples of solving the biharmonic and triharmonic equation over different multi-patch geometries are shown to demonstrate the potential of our IETI method.

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Testing tensor product Bézier surfaces for coincidence

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It is known that Bézier curves and surfaces may have multiple representations by different control polygons. The polygons may have a different number of control points and may even be disjoint. Up to our knowledge, Pekermann et al. [1] were the first to address the problem of testing two parametric polynomial curves for coincidence. Their approach is based on the reduction of the input curves into canonical irreducible form. They claimed that their approach can be extended for testing tensor product surfaces but gave no further detail.

We develop a new technique and provide a comprehensive solution to the problem of testing tensor product Bézier surfaces for coincidence. In [2] an algorithm for testing Bézier curves was proposed based on subdivision. There a partial solution to the problem of testing tensor product Bézier surfaces was presented. Namely, the case where the irreducible surfaces are of the same degree (n, m), $n, m \in \mathbb{N}$, was resolved under certain additional condition. The other case where one of the surfaces is of degree (n, m) and the other is of degree $(n + m, n + m)$ remained open.

We have implemented our algorithm for testing tensor product Bézier surfaces for coincidence using the Mathematica package. Experimental results and their analysis are presented.

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Approximating envelopes of evolving planar domains by arc splines

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joint work with Bert Jüttler

Sweeping a domain under a rigid body motion can be generalised by allowing the domain to change size or shape as it moves. We call such domains evolving domains, and the main challenge when considering this problem is to extract the envelope of the resulting volume.

In this talk, we characterize the envelopes of evolving worms: planar domains that are the cyclographic images of curve segments in the Minkowski space $\mathbb{R}^{2,1}$. The envelope is a subset of the cyclographic images of the boundary and singular curves on a surface in $\mathbb{R}^{2,1}$, which is generated as the curve segment (and the worm) evolves in time. We propose and compare two pairs of techniques for approximating the boundary and singular curves and the boundaries of their cyclographic images. In all four cases, the resulting superset of the envelope comprises circular arcs.

A free–form domain evolving in time is represented as a union of evolving worms, and its envelope is the subset of the envelopes of the evolving worms. The envelope can be extracted after effectively computing the intersections of the circular arcs using a line sweep algorithm.

Non-linear Moving Least Squares

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joint work with Juan Ruiz-Álvarez

In this talk, we present a data-dependent modification of the moving least squares (MLS) method, focusing on reducing the Gibbs phenomenon near discontinuities. Our approach introduces new weight functions that assign smaller weights to nodes located near discontinuities, while preserving the decay with distance from the point of approximation. This strategy allows for a sharper reconstruction of discontinuous data and minimizes unwanted oscillations. A key aspect of our method is the use of smoothness indicators—borrowed from the data-dependent WENO framework—to identify the nodes influenced by discontinuities. These indicators inform the construction of data-dependent weights that vary with both distance and local smoothness. In addition to modifying the weight functions, we also explore the role of the shape parameter in the approximation, adjusting it in a data-driven manner to further improve accuracy near discontinuities. We analyze the resulting approximant in terms of polynomial reproduction, accuracy, and smoothness, and assess its behavior with respect to diffusion and oscillations. Numerical experiments support our theoretical results and provide insight into the benefits of the proposed modifications.

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